UNA METODOLOGÍA PARA OPTIMIZAR EL DISEÑO DE JUNTAS EMPERNADAS

A METHODOLOGY TO OPTIMIZE THE DESIGN OF BOLTED JOINTS

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Abstract

The current paper presents a methodology to optimize the design of bolted joints under traction and shear loading in horizontal plates and pressure vessels. The MATLAB optimization tool is used to compute the ideal combination of bolt type and geometry, which leads to minimum joint cost, weight and maximum safety factor.

The methodology shows a cost of the joint-safety factor/mass-safety factor comparison which provide optimal options (curves) for the customer. These results allow choosing options according to the necessity, and decide if the cost or the mass of the bolted joint is more important in function of the safety factor with several correct solutions that can be adapted to the costumer's necessities.

Key Words— Genetic Algorithm (GA), Bolted Joints (BJ), Multi-objective optimization (MOO)

INTRODUCTION

Advanced and optimal designs are a common objective for all industries due to the necessity of reducing manufacturing cost and structure weight.

The design of bolted joints involves working with the variables i) the resistance of the bolt, ii) the joint plates, iii) the thickness of the structures to be join, where normally a diameter of a bolt

is chosen to check if the structure will be safe enough. Joint cylindrical plates normally are related to welding, but in this paper, cylindrical and horizontal bolted joints will be considered.

In the last decades a set of modern optimization tools which can solve engineering problems has been spreading. Examples of these tools are: genetic algorithms, simulated annealing, particle

swarm optimization, ant colony optimization, fuzzy optimization or neural-network based methods. In this paper, the problems will be solved by the genetic algorithms due to the ability for solving Multi-objective optimization (MOO) problems.

The methodology adopted in this paper aims to maximize the safety factor of the joint and minimize the weight of the elements needed to the joint (number of bolts, size of the bolt, and

This paper will solve three kinds of problems. First, horizontal plates of steel will be joined under traction and shear loading; then, horizontal plates of composite material will be jointed under the same conditions, and finally, steel cylinders under internal pressure will be joined using the same optimization tool.

A. GA optimization

Genetic Algorithms (GA) is one of the modern methods of optimization which can manage several objective functions to be maximized or minimized because GA is ability under given restrictions and limits, and GA is emerging to solve engineering problems.

In the last years, there has been a close relationship between the researchers studying evolutionary computation methods, which in many cases, GA has different meaning from its original approach from Holland.

B. Failure criteria

There are many failure criteria for making a safe bolted joint, where the material used is steel and there are countless theories available in the literature to predict a failure in all the elements of the joint under specific loads. (Budynas, Nisbert, & Screws, 2002)

In the same way, there are many failure criteria for composite materials (Orifici, Herszberg, & Thomson, 2008), having two categories to classify the failure criteria of the composite materials which are adopted in this paper, for more details the reader is referred to (Guillamet, Turon, Sebaey, & Costa, 2012)

The design of the bolted joint in this paper has been divided in two parts. The first one includes the design and selection of the bolt with all the necessary dimensions and values such as length of the bolt, step, height of the nut, threaded and non-threaded length of the grip, tension's stress area and nominal diameter´s area. The stress resistance is also chosen as a parameter inside de GA. The first part of the design of the bolted joints will predict the necessary bolt strength to support the load factors in the design. The design will include the design and selection of the thickness of the joint plates and its width as a function of the bolt diameter, and finally, the internal and external diameter of the cylinders under an internal pressure to be jointed are considered as parameters when we work with cylinders as elements to joint, and the thickness of the horizontal plates is considered as a parameter in the other case. In fact, the optimal selection of the bolt and its dimensions including thickness of the rest of elements involved is the goal in this category.

The second category includes the criteria associated to failure of the elements. That is possible by including mathematical expressions and some restrictions inside the GA´s program. These restrictions allow to the GA to provide solutions which satisfy the failure criteria. The stresses values generated by the internal pressure or loads i) tension and ii) shear. The stresses values must be lower than the strength of the bolts and the elements that make the bolted joint. In this category, the mechanical properties of the elements involved in the joint are clearly analyzed for all the cases.

According the design of the bolted joints developed in this paper, the formulation of the all the cases of study will be analyzed but, in the pressurized cylinders case, the load conditions must be just described.

B.1 For cylinder case

(a) The maximum pressure available (:

Figure 1: Stresses in a cylindrical body. Source: (American Society of Mechanical Engineers, 2013)

To determinate the admissible pressure (Pa) it is considered the maximum admissible strength (S), joint efficacy (Ej=0.65), thickness of the cylindrical body (t) and internal ratio (Ri).

For the tangential axis:

$$
P_a = \frac{S E_j t}{R_i + 0.6 t}
$$
 (1)

If: $t \leq R_i/2$

For the longitudinal axis

$$
P_a = \frac{2 \, S \, E_j \, t}{R_i - 0.4 \, t} \tag{2}
$$

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To determinate the stresses it is considered the working pressure (Pi), external ratio (Ro) and internal ratio (Ri).

(b) Longitudinal stress and tangential stress

$$
\sigma_t = \frac{{R_i}^2 P_i}{R_o^2 - R_i^2} \left(1 + \frac{{R_o}^2}{R^2} \right)
$$

$$
\sigma_L = \frac{{R_i}^2 P_i}{{R_o}^2 - {R_i}^2} \left(1 - \frac{{R_o}^2}{R^2}\right)
$$

(c) Area across the thickness (area_t)

Figure 2: Tangential (a) and Longitudinal (b) distribution of stresses. Source: (Budynas, Nisbert, & Screws, 2002)

$$
area_{\perp}t = 2 \pi (R_o^2 - R_i^2)
$$

Where the original area is multiplied by two because there will be two areas to joint under the load conditions. (F_{L}, F_{t})

For the present research the material selected for the study is a high speed steel which properties may be found on Table 1.

C. Bolted joint design analysis

Once we have the stresses (3)(4)/forces (1) (2) that will affect the joints (load conditions) in function of the internal pressure (for the case of cylindrical joint case), the maximum available pressure, and the traction and shear loads (for the horizontal plates joint case), the design to support these loads conditions begins.

First, the properties of the material i) maximum admissible strength (S), Young Modulus (E) and steel density (dens₁) to be used in the cylindric case is defined:

Source: Authors

Then, for the horizontal plates joint case, two materials will be joined under the same load conditions, one is the same high-speed steel with the properties shown in Table 1, and the other one is the composite material T300/5208 CFRP laminae (transversely isotropic material) with the following properties:

Source: Authors

The properties of the metric bolts: M8, M10, M12, M14, M16, M20 and M24 in different classes (strength) are shown in table 3.

Source: Authors

These bolts have been chosen due to the necessity to obtain commercial bolts (prize of the nut included considering a length of 200 mm). The tension's stress area is a property of each bolt, but there is a curve relationship between this and the diameter of the bolt, so the next equation is taken on the way to reduce one variable inside the GA.

Where the area is defined by:

 $A_t = 0.6799 d^2 + 0.0496 d - 6.4340$ (5)

Then, the stiffness of the bolts and joint needs to be calculated.

(a) Stiffness of the bolt (k_{b})

$$
k_b = \frac{A_d A_t E}{(A_d l_t) + (A_t l_d)}
$$
(6)

Where is the area of the nominal diameter of the bolt; is the threaded length in the grip, is the length of the grip (thickness of the cylinder + the thickness of the two joint plates) and is the length of the non-threaded portion in the grip.

(b) Stiffness of the joint (*km*)

Figure 4: Compression of an element with equivalent elastic properties represented by a trunk of a hollow cone. Here, " " represents the length of the grip. Source: (Budynas, Nisbert, & Screws, 2002)

$$
k = \frac{0.5774 \pi E d}{\ln \frac{(1.115t + D - d)(D + d)}{(1.155t + D + d)(D - d)}} \tag{7}
$$

If the two union members have the same Young modulus with symmetrical back-to-back trunks, then they act as two identical springs in series configuration. From equation (7), we know that $=$ /2. Using the grip as $= 2$, it is found that the spring ratio of the elements and alpha is 30° is given by:

$$
k_{m1} = \frac{0.5774 \pi E d}{2 \ln \left(\frac{5 (0.5774 l + 0.5 d)}{0.5774 l + 2.5 d} \right)}
$$
(8)

Then, the stiffness of the joint elements is given by:

$$
\frac{1}{k_m} = \frac{1}{k_{m1}} + \frac{1}{k_{m2}} + \frac{1}{k_{m3}} + \dots + \frac{1}{k_{mi}}
$$
(9)

(c) Stiffness constant (*C*)

$$
C = \frac{k_b}{k_b + k_m} \tag{10}
$$

Note that these values will change in function of the bolt chosen which are inside de GA.

(d) Preload of the bolt (F_i)

$$
F_i = 0.9 A_t S_p \tag{11}
$$

Where F_i is for permanent joints and is the strength probe (minimum).

For more details, the reader is referred to (Budynas, Nisbert, & Screws, 2002).

Finally, in Figure (5), it is represented graphically the description of the problem which is solved by the GA (cylindrical joint case).

Figure 5: Problem description (cylindrical joint case) Source: Authors

For the horizontal plates' joint case, is the same idea, joint two plates of HSS or composite material in the middle, with two joint plates of HSS (one above and one below) with one line of bolts in each horizontal plate as well.

D. Composite Failure Criteria

Mechanically as we defined, there is an explained approach to make a safety bolted joint with metal and with any other material using the equations (7), (8), and (9), but in this paper an analysis of the strength of all the plies in composite material with the laminate theory is necessary, for that reason the stress failure criteria will be adopted in this paper due to the simplicity of the theory, and its extensively use in the industry.

The composite material to be used in this paper is the T300/5208 CFRP laminae with a ply orientation $[0, 45, -45, 90]$, the properties required to solve the laminate theory are detailed on Table 2. This material is transversally isotropic, having one plane as an isotropic, and that will be useful in the methodology to calculate the stresses in the plies under the load conditions mentioned.

Five independent constants are required to characterize this material, considering the plane 2-3 as isotropic we have:

$$
E_{11},E_{22}=E_{33},\upsilon_{12}=\upsilon_{13},\upsilon_{23},G_{12}=G_{13}
$$

And with these values the compliance [S] and stiffness [C] matrixes are defined:

Then, the laminate theory is applied and the in-plane stiffness matrix [A], the coupling matrix [B], and the bending stiffness matrix [D] are required, for more information about the laminate theory the reader is referred to (Barbero, 2010).

Once we have the laminate theory, the maximum stress failure criteria is applied:

(a) Stress Failure Criteria.

Fiber: $\sigma_{11} \geq \sigma^u_{11T}$ or $\sigma_{11} \leq \sigma^u_{11C}$ Matrix: $\sigma_{22} \geq \sigma^u{}_{22T}$ or $\sigma_{22} \leq \sigma^u{}_{22C}$ Shear: $|\sigma_{12}| \geq \sigma_{12}^u$

The limit values are explained in Table 2.

The stress concentration produced by the presence of holes is considered by means of Stress concentration factor (Toubal, Karama, & Lorrain, 2005).

(b) Stress concentration factor ($K_{\pi/2}$)

$$
K_{\pi/2} = \frac{\sigma_y(r, 0)}{\sigma^{\infty}} = 1 + n
$$
 (12)

The variable is defined:

$$
n = \sqrt{2\left(\frac{E_{11}}{E_{22}} - 2v_{12}\right) + \frac{E_{11}}{G_{12}}}
$$
(13)

Before to apply the equation (13) it is necessary to compute the apparent properties of the consider laminate, due to this numerical approach needs the properties of the laminate, which are:

Source: Authors

Table 5 Apparent Properties of the laminate (values).

> $E_{xx} = 118560 MPa E_{yy} = 54090 MPa$ $E_{zz} = 12028 MPa$

Append

\n
$$
G_{yz} = 4719.8 \, MPa \, G_{zx} = 6077 \, MPa
$$
\nproperties.

\n
$$
G_{xy} = 15314 MPa
$$
\n
$$
v_{xy} = 0.20 \, v_{yz} = 0.38 \, v_{zx} = 0.03
$$
\nSources

\nSources

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Once we have the stress concentration factor at $K_{\pi/2}$ = 4.36 at (x=R), this value must be multiplied to the stress obtained in the stress analysis to have a very close approach to the real stresses in the most critical zone of the composite material, for more information the reader is referred to (Toubal, Karama, & Lorrain, 2005).

Figure 7: Failure hypothesis Source: (Whitney & Nuismer, 1974)

A. Constraint analysis

As mentioned above, the goal of this paper is to minimize the mass of the joint, the prize of it and maximize the safety factor. For that the GA used to solve this engineering problem is

"gamultiobj". This GA allow us to manage as many objective functions as we need to solve a specific problem under some engineering constraints, which will provide us a safe joint, for that the variables in the GA are the next, considering that the function to be called (N).

La aventura de descubrir

Table 6

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Variables in the GA program (cylindrical joint case).

Source: Authors

Table 7 Variables in the GA program (horizontal plate case).

Source: Authors

With these values in the program as variables, the GA solution will provide us the optimal solutions making a "Pareto Plot" to visualize all the combinations of the workable solutions. Next, the engineering restrictions needs to be defined in a function of the restriction function (*c*):

For more details about the Pareto plot, the reader is referred to (Instituto Uruguayo de Normas Técnicas, 2009).

(a) Traction in the bolt (c_1)

$$
\frac{\left(\frac{C\ n\ F_L}{Num} + F_i\right)}{A_t} \le S_p \tag{14}
$$

(b) Shear in the bolts (c_2)

$$
\frac{F_t n}{\left(\frac{0.577 \text{ Num } \pi D^2}{4}\right)} \le S_p \tag{15}
$$

(c) Bearing on the elements (Joint plates/ cylinder) $(c, 1)$

$$
\frac{F_L n}{2 N(3) D N u m} \le S \tag{16}
$$

(d) Bearing in the bolts (c_4)

$$
\frac{F_L n}{2 N(3) D N u m} \le S_p \tag{17}
$$

These constrains will provide us a safe joint to these of the possible failures mentioned, but there are other important parameters to consider especially in the cylindrical case. It is necessary to design the joint plates, and these plates need to be designed in function of the diameter of the bolt. The design of the joint plates includes the distance between the center of the holes length and wide (NBE EA-95, 1996). For that in the cylindrical recipient, a longitudinal constrain is generated, because we cannot have more bolts along the perimeter of the joint plate under these distances conditions.

Source: (Instituto Uruguayo de Normas Técnicas, 2009)

(e) Longitudinal restriction (c_s)

$$
1.5 D * Num \leq 2 \pi (R_0 + N(3)) \tag{18}
$$

The next constrain is defined by the admissible pressure in the cylinder case, this paper will show different curves for different internal pressures how is defined in the equation (1) and (2), but we cannot exceed the admissible pressure, that is not necessary to be done in the horizontal plates, for that in this paper, for the horizontal plates case, there will be a comparison between the joint steel plates and composite plates under the same load conditions.

(f) Internal pressure(P_i) (c_6)

A. Objective functions

How was said, the objective of this paper is to work with several main functions in the way to maximize or minimize according the engineering requirements, for that the functions used by the GA program are in a function of the main function *(f)*:

(a) Prize objective function (f_i)

To determinate the prize objective function it is considered the parameter $t_i = (Num - 1) * 3D + 4$ *D* (Length of the plates), $w_i = 300$ (mm) width of the plate to be joined, : 3.85 dollars/Kg for steel, $cost_2$: 150 dollars/m² (1 ply 0.125mm thick) of composite material

(a.1) Prize function for steel horizontal plates joint.

$$
f_1 = Num \, Prize + (2 (8 D t_1 N(3)) * dens_1 * cost_1) + (2 (w_1 t_1 N(5)) dens_1 * cost_1)
$$

(a.2) Prize function for composite material horizontal plates joint.

$$
f_1 = Num \, Prize + (2 (8 D t_1 N(3)) * dens_1 * cost_1) + (2 \left(w_1 t_1 \frac{N(5)}{0.125} \right) * cost_2/10^6)
$$

The result has 3 terms in the summation, the first one is the number of bolts obtained by the GA and multiplied by the correspond prize of each bolt detailed in Table 3, the second one is the volume of the two joint plates of steel (all the cases is the same. And, the third term is the calculation of the prize of the steel/composite material to be joined. For steel as a function of the density and for the composite material in function of the area of one ply of it.

(a.3) Prize function for cylindric joint.

In this case, the prize of the joint plates is not included because, this will add a kind of constant value to the total prize, that happens due to the cost of these plates are in function of the weight of them (here we only work with steel), so these values are in a range that be considered like a constant and will not represent a change in the results.

$$
f_1 = Num \, \text{Prize}
$$

(b) Safety factor of the joint $(f,)$

$$
f_2=\frac{1}{n}
$$

This is only a mathematical expression to be included in the GA to maximize the safety factor of the joint as we expect.

(c) Mass of the joint (f, x)

(c.1) Mass function for steel horizontal plates joint.

$$
f_{(3)} = \left[\left(\frac{\pi \,\mathrm{D}^2}{4} \, (l_b + l_t) \, Num \right) + 2 \left(\frac{1.5 \,\pi \,\mathrm{D}^2}{4} \, H \, Num \right) \right]. \, dens_1 \\ + \left(2 \left(8 \, D \, t_1 \, N(3) \right) * dens_1 \right) \\ + \left(2 \left(w_1 \, t_1 \, N(5) \right) dens_1 \right)
$$

(c.2) Mass function for composite material horizontal plates joint.

$$
f_{(3)} = \left[\left(\frac{\pi \, D^2}{4} \, (l_b + l_t) \, Num \right) + 2 \left(\frac{1.5 \, \pi \, D^2}{4} \, H \, Num \right) \right]. \, dens_1 \\ + \left(2 \left(8 \, D \, t_1 \, N(3) \right) * dens_1 \right) \\ + \left(2 \left(w_1 \, t_1 \, N(5) \right) dens_2 \right)
$$

(c.3) Mass function for cylindric case.

In this case the mass of the joint plate and the cylinders are not included neither, for the same reason mentioned below

$$
f_{(3)} = \left[\left(\frac{\pi\, {\rm D}^2}{4}\ (l_b + l_t)\ Num\right) + 2\left(\frac{1.5\ \pi\, {\rm D}^2}{4}\ H\ Num\right)\right].\ dens_1
$$

This procedure includes the same aspects of the objective function (), and the dimensions previously defined in the Figure 5, In fact the mass of all the elements of the bolted joint.

C. Variables' limits

The GA needs to work in range of values to evaluate the main functions inside values given by the programmer, so these values are added to the GA in the way to obtain values for a common dimension of the cylinder case / horizontal plates' case and those respective thicknesses.

Source: Authors

Source: Authors

A. Horizontal joint plate case.

The results for this case are shown after applying two loads, one perpendicular to the plates (F_1) , and one axial load (F_1) , the values are detailed in the Table 9.

> Table 11 Loads for horizontal joint plates.

Load conditions $F_L = 11000 (N)$ $F_t = 110000 (N)$

Source: Authors

For the joint of steel or composite material, the prize to obtain a correspond safety factor is:

Figure 9: Prize vs safety factor for metal and composite material. Source: Authors

As we can see, the values and curves obtained have big differences, that happens because we are considering all the elements which make the bolted jointed for each case, the joint plates, one above and one below of the horizontal plate (steel or composite), and the bolts, all of them are considered in the Figure 9 under the same load conditions. Having as results for the same safety factor, prizes around two or three times difference between both materials. Then, the analysis of the mass of all the joint is also analyzed under the same load conditions and the same considerations, assuming 300 millimeters width of the plate to join again (steel or composite) (w_1) .

Figure 10: Mass vs safety factor for metal and composite material. Source: Authors

Source: Authors

The values in the Table 12 correspond to a comparison between the highest and lowest safety factor value and the material to be joined in the same conditions, where the values of each Variable factor inside the GA are detailed (Table 7).

This information gives us a real and useful idea to decide how to design, where the designer or the costumer must take the decision if the budget or the mass of the structure has more importance, because only as an example, for the same safety factor (0.05), which is the biggest one, using steel, the bolted joint will have a mass of 224.13Kg with an inversion of 1687.2 dollars and, using the composite material it will have a mass of 65.93Kg with an inversion of 4458.5 dollars. The difference of mass and price are between two and three times, for that it is important to make the chose in function of the applications and the conditions of the bolted joint.

Then, how was mentioned before in the part II-D an analysis of the plies of the composite material is required to verify if there is not any ply overcharged. For that, in this analysis using the laminate theory and under the stress failure criteria (Barbero, 2010), only one load in the direction 2, detailed in the Figure 6, was applied.

 This load was the highest one, having the following plie's behavior.

Figure 11: Failure stress criteria vs Laminate thickness. Source: Authors

In the composite material analysis under 110000 Newtons in the direction 22 and according to the Figure 10, The composite material does not present any failure along the thickness, for that reason we can assume, in the direction 3, detailed in the Figure 6, and because the laminate behaves as a Quasi Isotropic material due to the plies orientation , there not will be failures under a lower load in that direction (load of 11000 Newtons) having not the necessity to analyze the plies in the direction 3.

In this case, to joint horizontal plates many loads conditions were tried in this part of the paper but, these combinations of loads can be supported for both materials without failures.

B. Cylindrical joint case

In the cylindrical joint case GA, the restriction of the internal pressure is included, so we cannot expect to obtain results if an input value above the internal pressure limit for that. In this paper, typical pressures common thicknesses of pressure Vessels were considered, but these values could change depending the application.

According to the results shown in Figure 12 and Figure 13, all the points generated are a possible solution of the problem for each pressure, which means that every point is a solution which satisfy the design requirements. In fact, for each point shown in the Figures 12 and 13, there are solutions for all the variables mentioned in the Tables 6 and 7, and these variables are making that, the all the possible combinations, give us the maximum safety factor, the minimum weight and the minimum prize of make it which are adjusted to a tendency line.

Next, as an example, the variables inside the GA are shown for all the internal pressures to obtain a safety factor between 0.25 and 0.26

Source: Authors

CONCLUSIONS

A methodology to design optimal bolted joints is proposed. The design variables considered are: bolt diameter, thickness of plates and joint geometry.

The objective functions considered are minimum weight, minimal cost and maximal safety factor.

The methodology works in the way to obtain the maximum or minimum value for each objective function in the GA, but satisfying the constrains given for the corresponding failure criteria theory.

The results aim to give to the designer/ costumer all the family of optimization values to analyze what constrain has more influence (weight/prize) for a specific application.

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